

Completing the square and turning point form

$$\pm a(x \pm b)^2 \pm c$$

The turning point form can be found either from a graph or factorising an expression by completing the square.

e.g. $y = x^2 - 4x + 6$

$$y = (x - 2)^2 - 4 + 6$$

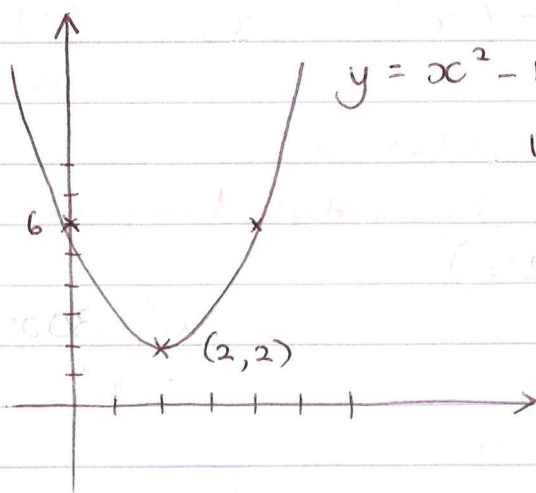
$$y = (x - 2)^2 + 2$$

+ Min



graph

t.p. (2, 2)



$$y = x^2 - 4x + 6$$

using

$$y = \pm a(x \pm b)^2 \pm c$$

$$y = +a(x - 2)^2 + 2$$

use (0, 6)

$$6 = +a(0 - 2)^2 + 2$$

$$6 = +a(-2)^2 + 2$$

$$6 = 4a + 2$$

$$4 = 4a$$

$$a = 1$$

the same!

so eqn of graph

$$y = (x - 2)^2 + 2$$

there are no factors of +6 that add to -4.
so complete the square

Remember

$$(x - 2)^2 = x^2 - 4x + 4$$

+4 too much

so we must - 4

$$(x - 2)^2 - 4 = x^2 - 4x$$

Another example

$$y = 30x - x^2$$

using graph to find
t.p. form

$$y = \pm a(x \pm b)^2 \pm c$$

$$y = -a(x - 15)^2 + 225$$

sub in (0,0)

$$0 = -a(0 - 15)^2 + 225$$

$$0 = -a(-15)^2 + 225$$

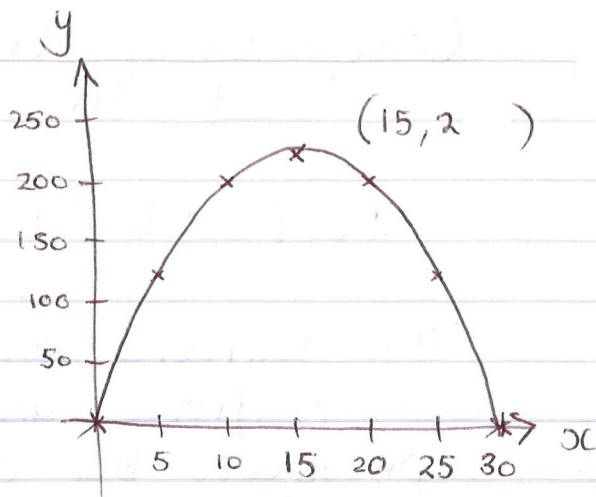
$$0 = -a \times 225 + 225$$

$$0 = -225a + 225$$

$$225a = 225$$

$$a = 1$$

$$y = -(x - 15)^2 + 225$$



or find t.p. form algebraically from $y = 30x - x^2$

$$y = 30x - x^2 = -x^2 + 30x \quad (\text{changed order})$$
$$= -(x^2 - 30x) \quad (-1 \text{ removed as common factor})$$

same

$$y = -(x^2 - 30x)$$

factorise by completing the square

$$y = -((x - 15)^2 - 225)$$

$$\rightarrow y = -(x - 15)^2 + 225$$

(multiplied
both terms
by -1)

$$x^2 - 30x + 225$$

too much

$$x^2 - 30x = (x - 15)^2 - 225$$